CHAPTER -11
ALGEBRAIC EXPRESSIONS

EXERCISE 11(A)

Question 1.
Separate the constants and variables from the following :

$$-7, 7 + x, 7x + yz, \sqrt{5}, \sqrt{xy}, \frac{3yz}{8}, 4.5y - 3x,$$

$$8 - 5, 8 - 5x, 8x - 5y \times p \text{ and } 3y^2z \div 4x$$

Solution:
Clearly constants are : -7, √5, 8 – 5

Variable are : $7 + x$, $7x + yz$, $\sqrt{xy}$, $\frac{3yz}{8}$, $4.5y$

$- 3x$

$8 - 5x$, $8x - 5y \times p$ and $3y^2 z \div 4x$

Question 2.
Write the number of terms in each of the following polynomials.
(i) $5x^2 + 3 \times ax$
(ii) $ax \div 4 - 7$
(iii) $ax - by + y \times z$
(iv) $23 + a \times b \div 2$.
Solution:

(i) $5x^2 + 3 \times ax = 5x^2 + 3ax$

∴ The number of terms in this polynomial = 2

(ii) $ax \div 4 - 7 = \dfrac{ax}{4} - 7$

∴ The number of terms in this polynomial = 2

(iii) $ax - by + y \times z = ax - by + yz$

∴ The number of terms in this polynomial = 3

(iv) $23 + a \times b \div 2 = 23 + \dfrac{ab}{2}$

∴ The number of terms in this Polynomial = 2

Question 3.
Separate monomials, binomials, trinomials and polynomials from the following algebraic expressions :

$8 - 3x, xy^2, 3y^2 - 5y + 8, 9x - 3x^2 + 15x^3 - 7,$

$3x \times 5y, 3x \div 5y, 2y \div 7 + 3x - 7$ and $4 - ax^2 + bx + y$

Solution:

Monomials are : $xy^2, 3x \times 5y, 3x \div 5y$;

Bionomials are : $8 - 3x$

Trinomials are : $3y^2 - 5y + 8, 2y \div 7 + 3x - 7$

Polynomials are : $8 - 3x, 3y^2 - 5y + 8, 9x - 3x^2 + 15x^3 - 7, 2y \div 7 + 3x -7, 4 - ax^2 + bx + y$

Question 4.
Write the degree of each polynomial given below :

(i) $xy + 7z$ (ii) $x^2 - 6x^3 + 8$

(iii) $y - 6y^2 + 5y^8$ (iv) $xyz - 3$

(v) $xy + yz^2 - zx^3$ (vi) $x^5y^7 - 8x^3 y^8 + 10x^4y^4 z^4$

Solution:
(i) degree = 2 (Polynomial is xy + 7z)
(ii) degree = 3 (Polynomial is $x^2 - 6x^3 + 8$ y)
(iii) degree = 8 (Polynomial is $y - 6y^2 + 5y^8$)
(iv) degree = 3 (Polynomial is xyz – 3)
(v) degree = 4 (Polynomial is $xy + yz^2 - xz^3$)
(vi) degree = 12 (Polynomial is $x^5y^7 - 8x^3y^8 + 10x^4, y^4z^4$)

Question 5.
Write the coefficient of :
(i) ab in 7abx ,
(ii) 7a in 7abx ;
(iii) $5x^2$ in $5x^2 - 5x$;
(iv) 8 in $a^2 - 8ax + a$;
(v) 4xy in $x^2 - 4xy + y^2$.
Solution:
(i) The coefficient of ab in 7abx = 7x
(ii) The coefficient of 7a in 7abx = bx
(iii) The coefficient of $5x^2$ in $5x^2 - 5x$ = 1
(iv) The coefficient of 8 in $a^2 - 8ax + a$ = – ax
(v) The coefficient of 4xy in $x^2 - 4xy + y^2$ = -1

Question 6.

In $\frac{5}{7}$ xy²z³, write the coefficient of

(i) 5   (ii) $\frac{5}{7}$ (iii) $5x$

(iv) xy^2 (v) z^3 (vi) xz^3

(vii) $5xy^2$ (viii) $\frac{1}{7}yz$ (ix) z

(x) yz^2 (xi) $5xyz$

Solution:

In $\frac{5}{7}$ xy²z³, Co-efficient of

(i) 5 is $\frac{1}{7}xy^2z^3$ (ii) $\frac{5}{7}$ is xy^2z^3

(iii) $5x$ is $\frac{1}{7}y^2z^3$ (iv) xy^2 is $\frac{5}{7}z^3$

(v) z^3 is $\frac{5}{7}xy^2$ (vi) xz^3 is $\frac{5}{7}y^2$

(vii) $5xy^2$ is $\frac{1}{7}z^3$ (viii) $\frac{1}{7}yz$ is $5xyz^2$

(ix) z is $\frac{5}{7}xy^2z^2$ (x) yz^2 is $\frac{5}{7}xy-z$

(xi) $5xyz$ is $\frac{1}{7}yz^2$

Question 7.

In each polynomial, given below, separate the like terms :

(i) $3xy, -4yx^2, 2xy^2, 2.5x^2y, -8yx, -3.2y^2x$ and x^2y

(ii) $y^2z^3, xy^2z^3, -5x^2yz, -4y^2z^3, -8xz^3y^2, 3x^2yz$ and $2z^3y^2$

Solution:

(i) Like terms are

$3xy, -8yx$; $-4yx^2, 2.5x^2y$ and x^2y ; $2xy^2$ and $-3.2y^2x$

(ii) $y^2z^3, -y^2z^3$ and $2z^3y^2$; xy^2z^3 and $-8xz^3y^2$; $-5x^2yz$ and $\Box x^2yz$

EXERCISE 11(B)

Question 1.
Evaluate :

(i) $-7x^2 + 18x^2 + 3x^2 - 5x^2$
(ii) $b^2y - 9b^2y + 2b^2y - 5b^2y$
(iii) $abx - 15abx - 10abx + 32abx.$
(iv) $7x - 9y + 3 - 3x - 5y + 8$
(v) $3x^2 + 5xy - 4y^2 + x^2 - 8xy - 5y^2$

Solution:

(i) $-7x^2 + 18x^2 + 3x^2 - 5x^2$
$$= 21x^2 - 12x^2$$
$$= 9x^2$$

(ii) $b^2y - 9b^2y + 2b^2y - 5b^2y$
$$= 3b^2y - 14b^2y$$
$$= -11b^2y$$

(iii) $abx - 15abx - 10abx + 32abx$
$$= 33abx - 25abx$$
$$= 8abx$$

(iv) $7x - 9y + 3 - 3x - 5y + 8$
$$= 7x - 3x - 9y - 5y + 3 + 8$$
$$= 4x - 14y + 11$$

(v) $3x^2 + 5xy - 4y^2 - 8xy - 5y^2$
$$= 3x^2 + 5xy - 8xy - 4y^2 - 5y^2$$
$$= 3x^2 - 3xy - 9y^2$$

Question 2.

Add :

(i) $5a + 3b$, $a - 2b$, $3a + 5b$

(ii) $8x - 3y + 7z$, $-4x + 5y - 4z$, $-x - y - 2z$

(iii) $3b - 7c + 10$, $5c - 2b - 15$, $15 + 12c + b$

(iv) $a - 3b + 3$; $2a + 5 - 3c$; $6c - 15 + 6b$

(v) $13ab - 9cd - xy$; $5xy$; $15cd - 7ab$; $6xy - 3cd$

(vi) $x^3 - x^2y + 5xy^2 + y^3$; $-x^3 - 9xy^2 + y^3$; $3x^2y + 9xy^2$

(vii) $a^6 - 4a^4 + 6a$; $5a^6 + 5a^4 + 6a$; $12a^6 - 10a$

(viii) $2ax - 6by + 4cz$, $4by - 14ax$, $9cz - 4ax - 6by$

Solution:

(i)
$$5a + 3b$$
$$a - 2b$$
$$3a + 5b$$
$$\overline{9a + 6b}$$

(ii)
$$8x - 3y + 7z$$
$$-4x + 5y - 4z$$
$$-x - y - 2z$$
$$\overline{3x + y + z}$$

(iii)
$$3b - 7c + 10$$
$$-2b + 5c - 15$$
$$+b + 12c + 15$$
$$\overline{2b + 10c + 10}$$

(iv)
$$a - 3b \qquad\quad + 3$$
$$2a \qquad\quad - 3c + 5$$
$$+ 6b + 6c - 15$$
$$\overline{3a + 3b + 3c - 7}$$

(v)
$$13ab - 9cd + xy$$
$$+ 5xy$$
$$-7ab + 15cd$$
$$- 3cd + 6xy$$
$$\overline{6ab + 3cd + 10xy}$$

(vi)
$$x^3 - x^2y + 5xy^2 + y^3$$
$$-x^3 \qquad\qquad - 9xy^2 + y^3$$
$$+ 3x^2y + 9xy^2$$
$$\overline{2x^2y + 5xy^2 + 2y^3}$$

Question 3.
Find the total savings of a boy who saves ₹ (4x – 6y) ; ₹ (6x + 2y) ; ₹ (4y – x) and ₹ (y – 2x) for four consecutive weeks.

Solution:
$$4x - 6y$$
$$6x + 2y$$
$$-x + 4y$$
$$-2x + y$$
$$\overline{7x + y}$$

∴ Total savings = ₹(7x + y)

Question 4.

(i) $4xy^2$ from $3xy^2$;
(ii) $-2x^2y + 3xy^2$ from $8x^2y$;
(iii) $3a-5b+c+2d$ from $7a-3b+c-2d$
(iv) $x^3 - 4x - 1$ from $3x^3 - x^2 + 6$
(v) $6a+3$ from a^3-3a^2+4a+1
(vi) $cab - 4cad - cbd$ from $3abc + 5bcd - cda$
(vii) $a^2 + ab + b^2$ from $4a^2 - 3ab + 2b^2$.

Solution:

(i) $3xy^2 - 4xy^2 = -xy^2$

(ii)
$$
\begin{array}{rr}
8x^2y & \\
-2x^2y & + 3xy^2 \\
+ & - \\
\hline
10x^2y & - 3xy^2
\end{array}
$$

(iii)
$$
\begin{array}{rrrrr}
7a & - 3b & + c & - 2d \\
3a & - 5b & + c & + 2d \\
- & + & - & - \\
\hline
4a & + 2b & & - 4d
\end{array}
$$

(iv)
$$
\begin{array}{rrrr}
3x^3 & - x^2 & & + 6 \\
x^3 & & - 4x & - 1 \\
- & & + & + \\
\hline
2x^3 & - x^2 & + 4x & + 7
\end{array}
$$

(v)
$$
\begin{array}{rrrr}
a^3 & - 3a^2 & + 4a & + 1 \\
& & + 6a & + 3 \\
& & - & - \\
\hline
a^3 & - 3a^2 & - 2a & - 2
\end{array}
$$

(vi)
$$
\begin{array}{rrr}
3abc & + 5bcd & - cda \\
+cab & - cbd & - 4cad \\
- & + & + \\
\hline
2abc & + 6bcd & + 3cad
\end{array}
$$

(vii)
$$
\begin{array}{rrr}
4a^2 & - 3ab & + 2b^2 \\
+ a^2 & + ab & + b^2 \\
- & - & - \\
\hline
3a^2 & - 4ab & + b^2
\end{array}
$$

Question 5.
(i) Take away – 3x³ + 4x² – 5x+ 6 from 3x³ – 4x² + 5x – 6
(ii) Take m² + m + 4 from -m² + 3m + 6 and the result from m² + m + 1.

Solution:

(i)
$$
\begin{array}{r}
3x^3 - 4x^2 + 5x - 6 \\
-3x^3 + 4x^2 - 5x + 6 \\
\hline
+ \quad - \quad\ \ + \quad\ - \\
\hline
6x^3 - 8x^2 + 10x - 12
\end{array}
$$

(ii)
$$
\begin{array}{r}
-m^2 + 3m + 6 \\
\pm m^2 \pm\ m \pm 4 \\
\hline
-2m^2 + 2m + 2
\end{array}
$$

A.T.Q.
$$
\begin{array}{r}
m^2 +\ m + 1 \\
-2m^2 + 2m + 2 \\
\hline
+ \quad\ - \quad\ - \\
\hline
3m^2 -\ m - 1
\end{array}
$$

Question 6.
Subtract the sum of $5y^2 + y - 3$ and $y^2 - 3y + 7$ from $6y^2 + y - 2$.
Solution:

$$
\begin{array}{r}
5y^2 + y - 3 \\
y^2 - 3y + 7 \\
\hline
6y^2 - 2y + 4
\end{array}
$$

$$
\begin{array}{r}
6y^2 + y - 2 \\
6y^2 - 2y + 4 \\
\hline
- \quad + \quad - \\
\hline
3y - 6
\end{array}
$$

Question 7.
What must be added to $x^4 - x^3 + x^2 + x + 3$ to obtain $x^4 + x^2 - 1$?
Solution:

$$
\begin{array}{r}
x^4 \quad\ + x^2 \quad\ - 1 \\
+x^4 - x^3 + x^2 + x + 3 \\
\hline
- \quad + \quad - \quad\ - \ - \\
\hline
x^3 \quad\ - x - 4
\end{array}
$$

Question 8.
(i) How much more than $2x^2 + 4xy + 2y^2$ is $5x^2 + 10xy - y^2$?
(ii) How much less $2a^2 + 1$ is than $3a^2 - 6$?

Solution:

(i)
$$\begin{array}{rrr} 5x^2 & + \ 10xy & - \ y^2 \\ +2x^2 & + \ \ 4xy & + \ 2y^2 \\ \hline \ - & \ - & \ - \\ \hline 3x^2 & + \ \ 6xy & - \ 3y^2 \end{array}$$

(ii)
$$\begin{array}{rr} 3a^2 & - \ 6 \\ +2a^2 & + \ 1 \\ \hline - & \ - \\ \hline a^2 & - \ 7 \end{array}$$

Question 9.

If x = 6a + 86 + 9c ; y = 2b – 3a – 6c and z = c – b + 3a ; find

(i) x + y + z

(ii) x – y + z

(iii) 2x – y – 3z

(iv) 3y – 2z – 5x

Solution:

(i)
$$\begin{array}{rrrr} x = & 6a & + \ 8b & + \ 9c \\ y = & -3a & + \ 2b & - \ 6c \\ z = & +3a & - \ b & + \ c \\ \hline \text{Adding } x+y+z = & 6a & + \ 9b & +4c \end{array}$$

(ii) $x-y+z = (6a+8b+9c)-(2b-3a-6c)$
$$+ (c-b+3a)$$
$$= 6a+8b+9c-2b+3a+6c+c-b+3a$$
$$= 6a+3a+3a+8b-2b-b+9c+6c+c$$
$$= 12a+5b+16c$$

(iii) $2x-y-3z = 2(6a+8b+9c)-(2b-3a-6c)$
$$-3(c-b+3a)$$
$$=12a+16b+18c-2b+3a+6c-3c+3b-9a$$
$$=12a+3a-9a+16b+3b-2b+18c+6c-3c$$
$$= 6a+17b+21c$$

(iv) $3y-2z-5x = 3(2b-3a-6c)-2(c-b+3a)-$
$$5(6a+8b+9c)$$
$$=6b-9a-18c-2c+2b-6a-30a-40b-45c$$
$$=-9a-6a-30a+6b+2b-40b-18c-2c-45c$$
$$= -45a-32b-65c$$

Question 10.
The sides of a triangle are $x^2 - 3xy + 8$, $4x^2 + 5xy - 3$ and $6 - 3x^2 + 4xy$. Find its perimeter.
Solution:

$$\text{Required perimeter} = \text{Sum of three sides}$$
$$= x^2 - 3xy + 8 + 4x^2 + 5xy - 3 + 6 - 3x^2 + 4xy$$
$$= x^2 + 4x^2 - 3x^2 - 3xy + 5xy + 4xy + 8 - 3 + 6$$
$$= 2x^2 + 6xy + 11$$

Question 11.
The perimeter of a triangle is $8y^2 - 9y + 4$ and its two sides are $3y^2 - 5y$ and $4y^2 + 12$. Find its third side.
Solution:

$$\text{Perimeter of the triangle} = \text{Sum of three sides}$$
$$= 8y^2 - 9y + 4$$
$$\text{Sum of two sides} = 3y^2 - 5y + 4y^2 + 12$$
$$= 7y^2 - 5y + 12$$
$$\therefore (8y^2 - 9y + 4) - (7y^2 - 5y + 12)$$
$$= 8y^2 - 9y + 4 - 7y^2 + 5y - 12$$
$$= y^2 - 4y - 8$$
$$\text{Hence third side} = y^2 - 4y - 8$$

Question 12.
The two adjacent sides of a rectangle are $2x^2 - 5xy + 3z^2$ and $4xy - x^2 - z^2$. Find its perimeter.
Solution:

$$\text{Adjacent sides of a rectangle are}$$
$$2x^2 - 5xy + 3z^2 \text{ and } 4xy - x^2 - z^2$$
$$\therefore \text{Perimeter} = 2(2x^2 - 5xy + 3z^2 + 4xy - x^2 - z^2)$$
$$= 4x^2 - 10xy + 6z^2 + 8xy - 2x^2 - 2z^2$$
$$= 2x^2 - 2xy + 4z^2$$

Question 13.
What must be subtracted from $19x^4 + 2x^3 + 30x - 37$ to get $8x^4 + 22x^3 - 7x - 60$?
Solution:

The required result will be
$$(19x^4 + 2x^3 + 30x - 37) - (8x^4 + 22x^3 - 7x - 60)$$
$$= 19x^4 + 2x^3 + 30x - 37 - 8x^4 - 22x^3 + 7x + 60$$
$$= 11x^4 - 20x^3 + 37x + 23$$

Question 14.
How much smaller is 15x – 18y + 19z than 22x – 20y – 13z + 26 ?
Solution:
The required result is
(22x – 20y – 13z + 26) – (15x – 18y + 19z)
= 22x – 20y – 13z + 26 – 15x + 18y – 19z
= 7x – 2y – 32z + 26

Question 15.
How much bigger is 15x²y² – 18xy² – 10x²y than -5x² + 6x²y – 7xy ?
Solution:
The required result,
$$(5x^2y^2 - 18xy^2 - 10x^2y) - (-5x^2 + 6x^2y - 7xy)$$
$$= 5x^2y^2 - 18xy^2 - 10x^2y + 5x^2 - 6x^2y + 7xy$$
$$= 5x^2y^2 - 18xy^2 - 16x^2y + 5x^2 + 7xy$$

Question 1.

Multiply :

(i) $8ab^2$ by $-4a^3b^4$

(ii) $\dfrac{2}{3}ab$ by $-\dfrac{1}{4}a^2b$

(iii) $-5cd^2$ by $-5cd^2$.

(iv) $4a$ and $(6a + 7)$

(v) $-8x$ and $(4 - 2x - x^2)$

(vi) $2a^2 - 5a - 4$ and $-3a$.

(vii) $x + 4$ by $x - 5$

(viii) $5a - 1$ by $7a - 3$

(ix) $12a + 5b$ by $7a - b$

(x) $x^2 + x + 1$ by $1 - x$

(xi) $2m^2 - 3m - 1$ and $4m^2 - m - 1$

(xii) a^2, ab and b^2

(xiii) abx, $-3a^2x$ and $7b^2x^3$

(xiv) $-3bx$, $-5xy$ and $-7b^3y^2$

(xv) $\left(-\dfrac{3}{2}x^5y^3\right)$ and $\left|\left(\dfrac{4}{9}a^2x^3y\right)\right.$

(xvi) $\left(-\dfrac{2}{3}a^7b^2\right)$ and $\left(-\dfrac{9}{4}ab^5\right)$

(xvii) $(2a^3 - 3a^2b)$ and $\left(-\dfrac{1}{2}ab^2\right)$

(xviii) $\left(2x + \dfrac{1}{2}y\right)$ and $\left(2x - \dfrac{1}{2}y\right)$

Solution:

(i) $8ab^2 \times -4a^3b^4 = (8 \times -4)(ab^2 \times a^3b^4)$

$\qquad\qquad\qquad\quad = -32a^{1+3} \cdot b^{2+4}$

$\qquad\qquad\qquad\quad = -32a^4b^6$

(ii) $\dfrac{2}{3}ab \times -\dfrac{1}{4}a^2b = \left(\dfrac{2}{3} \times \dfrac{-1}{4}\right)(ab \times a^2b)$

$\qquad\qquad\qquad\qquad = -\dfrac{1}{6}a^{1+2} \cdot b^{1+1}$

$\qquad\qquad\qquad\qquad = -\dfrac{1}{6}a^3b^2$

(iii) $-5cd^2 \times -5cd^2 = (-5 \times -5)(cd^2 \times cd^2)$

$\qquad\qquad\qquad\qquad = 25c^{1+1}d^{2+2}$

$\qquad\qquad\qquad\qquad = 25c^2d^4$

(iv) $4a\,(6a + 7)$

$\qquad\quad = 4a \times 6a + 4a \times 7$

$\qquad\quad = 24a^2 + 28a$

(v) $-8x\,(4 - 2x - x^2)$

$\qquad\quad = -8x \times 4 - 8x \times -2x - 8x \times -x^2$

$\qquad\quad = -32x + 16x^2 + 8x^3$

(vi) $-3a\,(2a^2 - 5a - 4)$

$\qquad\quad = -3a \times 2a^2 - 5a \times -3a - 4$

$\qquad\qquad\qquad\qquad\qquad\qquad \times -3a$

$\qquad\quad = -6a^3 + 15a^2 + 12a$

(vii) $(x+4)(x-5) = x(x-5) + 4\,(x-5)$

$\qquad\qquad\qquad = x^2 - 5x + 4x - 20$

$\qquad\qquad\qquad = x^2 - x - 20$

(viii) $(5a - 1)(7a - 3)$

$\qquad\quad = 5a\,(7a - 3) - 1\,(7a - 3)$

$\qquad\quad = 35a^2 - 15a - 7a + 3$

$\qquad\quad = 35a^2 - 22a + 3$

(ix) $(12a + 5b)(7a - b) = 12a\,(7a - b) + 5b\,(7a - b)$

$\qquad\quad = 84a^2 - 12ab + 35ab - 5b^2$

$\qquad\quad = 84a^2 + 23ab - 5b^2$

(x) $(x^2+x+1)(1-x) = 1\,(x^2+x+1) - x(x^2+x+1)$

$\qquad\quad = x^2+x+1 -x^3-x^2-x.$

$\qquad\quad = 1-x^3$

(xi) $(2m^2 - 3m - 1)(4m^2 - m - 1)$

$= 2m^2(4m^2 - m - 1) - 3m(4m^2 - m - 1) - 1(4m^2 - m - 1)$

$= 8m^4 - 2m^3 - 2m^2 - 12m^3 + 3m^2 + 3m - 4m^2 + m + 1$

$= 8m^4 - 14m^3 - 6m^2 + 3m^2 + 4m + 1$

$= 8m^4 - 14m^3 - 3m^2 + 4m + 1$

(xii) $a^2 \times ab \times b^2 \qquad\qquad = a^{2+1}.b^{1+2}$

$\qquad\qquad\qquad\qquad\qquad\qquad = a^3b^3$

(xiii) $abx \times -3a^2x \times 7b^2x^3$

$\qquad = (-3 \times 7)(a \times a^2)(b \times b^2)(x \times x \times x^3)$

$\qquad = -21a^3b^3x^5$

(xiv) $-3bx \times -5xy \times -7b^3y^2$

$\qquad = (-3 \times -5 \times -7)(b \times b^3)(x \times x)(y \times y^2)$

$\qquad = -105\, b^4x^2y^3$

(xv) $\left(-\dfrac{3}{2}x^5y^3\right)\left(\dfrac{4}{9}a^2x^3y\right)$

$\qquad\qquad = \left(-\dfrac{3}{2} \times \dfrac{4}{9}\right)(a^2)(x^5 \times x^3)(y^3 \times y)$

(xvi) $\left(-\dfrac{2}{3}a^7b^2\right)\left(-\dfrac{9}{4}ab^5\right)$

$\qquad\qquad = \left(-\dfrac{2}{3} \times \dfrac{-9}{4}\right)(a^7 \times a)(b^2 \times b^5)$

$\qquad\qquad = \dfrac{3}{2}a^8b^7$

(xvii) $(2a^3 - 3a^2b)\left(-\dfrac{1}{2}ab^2\right)$

$\qquad\qquad = -\dfrac{1}{2}ab^2(2a^3 - 3a^2b)$

$\qquad\qquad = 2a^3 \times -\dfrac{1}{2}ab^2 - 3a^2b \times -\dfrac{1}{2}ab^2$

$\qquad\qquad = -a^4b^2 + \dfrac{3}{2}a^3b^3$

(xviii) $\left(2x + \dfrac{1}{2}y\right)\left(2x - \dfrac{1}{2}y\right)$

$\qquad\qquad = 2x\left(2x - \dfrac{1}{2}y\right) + \dfrac{1}{2}y\left(2x - \dfrac{1}{2}y\right)$

$= 4x^2 - xy + xy - \dfrac{1}{4}y^2$

$= 4x^2 - \dfrac{1}{4}y^2$

Multiply :

(i) $5x^2 - 8xy + 6y^2 - 3$ by $-3xy$

(ii) $3 - \dfrac{2}{3}xy + \dfrac{5}{7}xy^2 - \dfrac{16}{21}x^2y$ by $-21x^2y^2$

(iii) $6x^3 - 5x + 10$ by $4 - 3x^2$

(iv) $2y - 4y^3 + 6y^5$ by $y^2 + y - 3$

(v) $5p^2 + 25pq + 4q^2$ by $2p^2 - 2pq + 3q^2$

Solution:

(i) $5x^2 - 8xy + 6y^2 - 3 \times -3xy$

$= 15x^3y^3 + 24x^2y^2 - 18xy^3 + 9xy$

(ii) $3 - \dfrac{2}{3}xy + \dfrac{5}{7}xy^2 - \dfrac{16}{21}x^2y$

$\times \qquad -21x^2y^2$

$-63x^2y^2 + 14x^3y^3 - 15x^3y^4 + 16x^4y^3$

(iii)

$$
\begin{array}{r}
6x^3 \quad - 5x \quad + 10 \\
\times \quad 4 \quad - 3x^2 \\
\hline
24x^3 \quad - 20x \quad + 40 \\
- 18x^5 + 15x^3 - 30x^2 \\
\hline
- 18x^5 + 39x^3 - 30x^2 - 20x + 40
\end{array}
$$

(iv)

$$
\begin{array}{r}
2y - 4y^3 + 6y^5 \\
\times \quad y^2 + y - 3 \\
\hline
2y^3 - 4y^5 + 6y^7 \\
+ 2y^2 - 4y^4 + 6y^6 \\
- 6y + 12y^3 - 18y^5 \\
\hline
\end{array}
$$

$6y^7 + 6y^6 - (4 + 18)y^5 - 4y^4 + (2 + 12)y^3 + 2y^2 - 6y$

$= 6y^7 + 6y^6 - 22y^5 - 4y^4 + 14y^3 + 2y^2 - 6y$

(v)

$$
\begin{array}{r}
5p^2 + 25pq + 4q^2 \\
\times \quad 2p^2 - 2pq + 3q^2 \\
\hline
10p^4 + 50p^3q + 8p^2q^2 \\
- 10p^3q - 50p^2q^2 - 8pq^3 \\
+ 15p^2q^2 + 75pq^3 + 12q^4 \\
\hline
10p^4 + 40p^3q - 27p^2q^2 + 67pq^3 + 12q^4
\end{array}
$$

Question 3.
Simplify :
(i) (7x – 8) (3x + 2)
(ii) (px – q) (px + q)
(iii) (5a + 5b – c) (2b – 3c)
(iv) (4x – 5y) (5x – 4y)
(v) (3y + 4z) (3y – 4z) + (2y + 7z) (y + z)

Solution:

(i) $(7x - 8)(3x + 2) = 7x(3x + 2) - 8(3x + 2)$

$= 21x^2 + 14x - 24x - 16 = 21x^2 - 10x - 16$

(ii) $(px - q)(px + q) = px(px + q) - q(px + q)$

$= p^2x^2 + pxq - pqx - q^2 = p^2x^2 - q^2$

(iii) $(5a + 5b - c)(2b - 3c)$

$= 5a(2b - 3c) + 5b(2b - 3c) - c(2b - 3c)$

$= 10ab - 15ac + 10b^2 - 15bc - 2bc + 3c^2$

$= 10ab + 10b^2 - 17bc - 15ac + 3c^2$

(iv) $(4x - 5y)(5x - 4y)$

$= 4x(5x - 4y) - 5y(5x - 4y)$

$= 20x^2 - 16xy - 25xy + 20y^2$

$= 20x^2 - 41xy + 20y^2$

(v) $(3y + 4z)(3y - 4z) + (2y + 7z)(y + z)$

$= 3y(3y - 4z) + 4z(3y - 4z) + 2y(y + z) + 7z(y + z)$

$= 9y^2 - 12yz + 12yz - 16z^2 + 2y^2 + 2yz + 7yz + 7z^2$

$= (9 + 2)y^2 + (-12 + 12 + 2 + 7)yz + (-16 + 7)z^2$

$= 11y^2 + 9yz - 9z^2$

Question 4.
The adjacent sides of a rectangle are $x^2 - 4xy + 7y^2$ and $x^3 - 5xy^2$. Find its area.
Solution:

$$\text{Reqd. area} = (x^2 - 4xy + 7y^2)(x^3 - 5\ xy^2)$$
$$= x^2(x^3 - 5xy^2) - 4xy(x^3 - 5xy^2) + 7y^2(x^3 - 5xy^2)$$
$$= x^5 - 5x^3y^2 - 4x^4y + 20x^2y^3 + 7x^3y^2 - 35xy^4$$
$$= x^5 + (7 - 5)x^3y^2 - 4x^4y + 20x^2y^3 - 35xy^4$$
$$= x^5 + 2x^3y^2 - 4x^4y + 20x^2y^3 - 35xy^4$$
$$= (x^5 - 4x^4y + 2x^3y^2 + 20x^2y^3 - 35xy^4) \text{ sq. unit.}$$

Question 5.
The base and the altitude of a triangle are $(3x - 4y)$ and $(6x + 5y)$ respectively. Find its area.
Solution:

Reqd. Area $= \frac{1}{2}$ (base) x (altitude)

$$= \frac{1}{2} (3x - 4y)(6x + 5y)$$

$$= \frac{1}{2}(18x^2 + 15xy - 24xy - 20y^2)$$

$$= \frac{1}{2}(18x^2 - 9xy - 20y^2) \text{ sq. unit.}$$

Question 6.
Multiply $-4xy^3$ and $6x^2y$ and verify your result for $x = 2$ and $y = 1$.
Solution:

$$(-4xy^3) \times (6x^2y) = (-4 \times 6)(x \times x^2)(y^3 \times y)$$
$$= -24x^3y^4$$

For $x = 2$ and $y = 1$

$$(-4xy^3) \times (6x^2y) = (-4 \times 2 \times 1^3) \times (6 \times 2^2 \times 1)$$

$$= (-8) \times 24 = -192$$

And, $-24x^3y^4 = -24 \times 2^3 \times 1^4$

$$= -24 \times 8 \times 1 = -192$$

$\therefore$ For $x = 2$ and $y = 1$, it is verified that

$$(-4xy^3) \times (6x^2y) = -24x^3y^4$$

Question 7.

Find the value of $(3x^3) \times (-5xy^2) \times (2x^2yz^3)$ for $x = 1$, $y = 2$ and $z = 3$.

Solution:

For $x = 1$, $y = 2$ and $z = 3$

$(3x^3) \times (-5xy^2) \times (2x^2yz^3)$

$(3 \times 1^3) \times (-5 \times 1 \times 2^2) \times (2 \times 1^2 \times 2 \times 3^3)$

$3 \times (-5 \times 4) \times (2 \times 1 \times 2 \times 27)$

$3 \times (-20) \times 108 = -6480$

Question 8.

Evaluate $(3x^4y^2)(2x^2y^3)$ for $x = 1$ and $y = 2$.

Solution:

$(3x^4y^2)(2x^2y^3)$

$(3 \times 1^4 \times 2^2) \times (2 \times 1^2 \times 2^3)$

$(3 \times 1 \times 4) \times (2 \times 1 \times 8)$

$= 12 \times 16 = 192$

Question 9.

Evaluate $(x^5) \times (3x^2) \times (-2x)$ for $x = 1$.

Solution:

For $x = 1$

$(x^5) \times (3x^2) \times (-2x)$

$(1^5) \times (3 \times 1^2) \times (-2 \times 1)$

$1 \times 3 \times (-2) = -6$

Question 10.

If $x = 2$ and $y = 1$; find the value of $(-4x^2y^3) \times (-5x^2y^5)$.

Solution:

For $x = 2$ and $y = 1$

$(-4x^2y^3) \times (-5x^2y^5)$

$(-4 \times 2^2 \times 1^3) \times (-5 \times 2^2 \times 1^5)$

$(-4 \times 4 \times 1) \times (-5 \times 4 \times 1)$

$-16 \times -20 = 320$

Question 11.

Evaluate:

(i) $(3x - 2)(x + 5)$ for $x = 2$.

(ii) $(2x - 5y)(2x + 3y)$ for $x = 2$ and $y = 3$.

(iii) $xz (x^2 + y^2)$ for $x = 2$, $y = 1$ and $z = 1$.

Solution:

(i) For $x = 2$

$(3x - 2)(x + 5)$

$(3 \times 2 - 2)(2 + 5)$

$(6 - 2) \times 7$

$4 \times 7 = 28$

(ii) For $x = 2$ and $y = 1$

$xy^2(x - 5y) + 1$

$2 \times 1^2(2 - 5 \times 1) + 1$

$2 \times (2 - 5) + 1$

$2 \times (-3) + 1$

$-6 + 1 = -5$

(iii) For $x = 2$, $y = 1$ and $z = 1$

$xz(x^2 + y^2)$

$2 \times 1(2^2 + 1^2)$

$2(2 + 1)$

$= 2 \times 3 = 6$

Question 12.

Evaluate:

(i) $x(x - 5) + 2$ for $x = 1$.

(ii) $xy^2(x - 5y) + 1$ for $x = 2$ and $y = 1$.

(iii) $2x(3x - 5) - 5(x - 2) - 18$ for $x = 2$.

Solution:

(i) For $x = 1$

$x(x - 5) + 2$

$1(1 - 5) + 2$

$-4 + 2 = -2$

(ii) For $x = 2$ and $y = 1$

$xy^2(x - 5y)$

$2 \times 1^2(2 - 5 \times 1)$

$2 \times (2 - 5)$

$2 \times (-3) = -6$

(iii) For $x = 2$

$2x(3x - 5) - 5(x - 2) - 18$

$2 \times 2(3 \times 2 - 5) - 5(2 - 2) - 18$

$4(6 - 5) - 5 \times 0 - 18$

$4 - 18 = -14$

Question 13.
Multiply and then verify :
$-3x^2y^2$ and $(x - 2y)$ for $x = 1$ and $y = 2$.
Solution:

$$(-3x^2y^2) \times (x - 2y)$$
$$= (-3x^2y^2) \times (x) - (-3x^2y^2)(2y)$$
$$= -3x^3y^2 + 6x^2y^3$$
$$= 6x^2y^3 - 3x^3y^2$$

For $x = 1$ and $y = 2$

$$(-3x^2y^2) \times (x - 2y)$$
$$= (-3 \times 1^2 \times 2^2) \times (1 - 2 \times 2)$$
$$= (6 \times 1 \times 8) - (3 \times 1 \times 4)$$
$$= 48 - 12 = 36$$

$\therefore$ For $x = 1$ and $y = 2$, it is verified that,
$$(-3x^2y^2) \times (x - 2y) = 6x^2y^3 - 3x^3y^2$$

Question 14.
Multiply:
(i) $2x^2 - 4x + 5$ by $x^2 + 3x - 7$
(ii) $(ab - 1)(3 - 2ab)$

(i) $2x^2 - 4x + 5$ by $x^2 + 3x - 7$
$$(2x^2 - 4x + 5) \times (x^2 + 3x - 7)$$
$$2x^2(x^2 + 3x - 7) - 4x(x^2 + 3x - 7) + 5(x^2 + 3x - 7)$$
$$2x^4 + 6x^3 - 14x^2 - 4x^3 - 12x^2 + 28x + 5x^2 + 15x - 35$$
$$2x^4 + 6x^3 - 4x^3 - 14x^2 - 12x^2 + 5x^2 + 28x + 15x - 35$$
$$2x^4 + 2x^3 - 21x^2 + 43x - 35$$

(ii) $(ab - 1)(3 - 2ab)$
$$ab(3 - 2ab) - 1(3 - 2ab)$$
$$3ab - 2a^2b^2 - 3 + 2ab$$
$$-2a^2b^2 + 5ab - 3$$
$$2a^2b^2 - 5ab + 3$$

Question 15.
Simplify : $(5 – x)(6 – 5x)(2 -x)$.
Solution:

$(5 – x)(6 – 5x)(2 – x)$

$[(5 – x)(6 – 5x)](2 – x)$

$[5(6 – 5x) – x(6 – 5x)](2 – x)$

$[30 – 25x – 6x + 5x^2](2 – x)$

$(5x^2 – 31x + 30)(2 – x)$

$2(5x^2 – 31x + 30) – x(5x^2 – 31x + 30)$

$10x^2 – 62x + 60 – 5x^3 + 31x^2 – 30x$

$-5x^3 + 10x^2 + 31x^2 – 62x – 30x + 60$

$-5x^3 + 41x^2 – 92x + 60$

Question 1.

Divide :

(i) $-70a^3$ by $14a^2$

(ii) $24x^3y^3$ by $-8y^2$

(iii) $15a^4b$ by $-5a^3b$

(iv) $-24x^4d^3$ by $-2x^2d^5$

(v) $63a^4b^5c^6$ by $-9a^2b^4c^3$

(vi) $8x - 10y + 6c$ by 2.

(vii) $15a^3b^4 - 10a^4b^3 - 25a^3b^6$ by $-5a^3b^2$

(viii) $-14x^6y^3 - 21x^4y^5 + 7x^5y^4$ by $7x^2y^2$

(ix) $a^2 + 7a + 12$ by $a + 4$

(x) $x^2 + 3x - 54$ by $x - 6$

(xi) $12x^2 + 7xy - 12y^2$ by $3x + 4y$

(xii) $x^6 - 8$ by $x^2 - 2$

(xiii) $6x^3 - 13x^2 - 13x + 30$ by $2x^2-x-6$

(xiv) $4a^2 + 12ab + 9b^2 - 25c^2$ by $2a+3b+5c$.

(xv) $16+8x+x^6-8x^3-2x^4+x^2$ by $x + 4 - x^3$

Solution:

(i)
$$\frac{-70a^3}{14a^2} = \left(\frac{-70}{14}\right)\left(\frac{a^3}{a^2}\right)$$
$$= -5a^{3-2}$$
$$= -5a$$

(ii) $\dfrac{24x^3y^3}{-8y^2} = \left(\dfrac{24}{-8}\right)(x^3)\left(\dfrac{y^3}{y^2}\right)$

$= -3x^3y^{3-2}$

$= -3x^3y$

(iii) $\dfrac{15a^4b}{-5a^3b} = \left(\dfrac{15}{-5}\right)\left(\dfrac{a^4}{a^3}\right)\left(\dfrac{b}{b}\right)$

$= -3a^{4-3}b^{1-1}$

$= -3a\ b^0$

$= -3a \times 1 \quad (\because b^0 = 1)$

$= -3a$

(iv) $\dfrac{-24x^4d^3}{-2x^2d^5} = \left(\dfrac{-24}{-2}\right)\left(\dfrac{x^4}{x^2}\right)\left(\dfrac{d^3}{d^5}\right)$

$= 12x^{4-2}d^{3-5} = 12x^2d^{-2}$

$= \dfrac{12x^2}{d^2}$

(v) $\dfrac{63a^4b^5c^6}{-9a^2b^4c^3} = \left(\dfrac{63}{-9}\right)\left(\dfrac{a^4}{a^2}\right)\left(\dfrac{b^5}{b^4}\right)\left(\dfrac{c^6}{c^3}\right)$

$= -7a^{4-2} \cdot b^{5-4} \cdot c^{6-3}$

$= -7a^2bc^3$

(vi) $\dfrac{8x - 10y + 6c}{2}$

$= \dfrac{8x}{2} - \dfrac{10y}{2} + \dfrac{6c}{2}$

$= 4x - 5y + 3c$

(vii) $\dfrac{15a^3b^4 - 10a^4b^3 - 25a^3b^6}{-5a^3b^2}$

$$= \frac{15a^3b^4}{-5a^3b^2} - \frac{10a^4b^3}{-5a^3b^2} - \frac{25a^3b^6}{-5a^3b^2}$$

$$= -3b^{4-2} + 2a^{4-3}b^{3-2} + 5b^{6-2}$$

$$= -3b^2 + 2ab + 5b^4$$

(viii) $\dfrac{-14x^6y^3 - 21x^4y^5 + 7x^5y^4}{7x^2y^2}$

$$= \frac{-14x^6y^3}{7x^2y^2} - \frac{21x^4y^5}{7x^2y^2} + \frac{7x^5y^4}{7x^2y^2}$$

$$= -2x^{6-2}y^{3-2} - 3x^{4-2}y^{5-2} + x^{5-2}y^{4-2}$$

$$= -2x^4y - 3x^2y^3 + x^3y^2$$

(ix)
```
            a + 3
a+4 ) a² +7a +12
      a² +4a
      -   -
      _____
            3a + 12
            3a + 12
            -    -
      _____
               ×
      _____
```

∴ Answer $= a + 3$

(x)
```
            x + 9
x-6 ) x² +3x -54
      x² -6x
      -   +
      _____
            9x - 54
          + 9x - 54
          -    +
      _____
               ×
      _____
```

∴ Answer $= x + 9$

(xi)

$$3x+4y \overline{)12x^2+7xy-12y^2}(\ 4x-3y$$

$$12x^2+16xy$$

$$\underline{\quad -\quad -\quad}$$

$$-9xy-12y^2$$

$$-9xy-12y^2$$

$$\underline{\quad +\quad +\quad}$$

$$\times$$

∴ Answer = $4x - 3y$

(xii)

$$x^2-2 \overline{)\ x^6-8\quad}(\ x^4+2x^2+4$$

$$x^6\quad -2x^4$$

$$\underline{-\quad +\quad}$$

$$2x^4 - 8$$

$$2x^4\quad\ -4x^2$$

$$\underline{-\qquad +\quad}$$

$$4x^2 - 8$$

$$4x^2 - 8$$

$$\underline{-\quad +\quad}$$

$$\times$$

∴ Answer = x^4+2x^2+4

(xiii)

$$2x^2-x-6 \overline{)6x^3-13x^2-13x+30}(\ 3x-5$$

$$6x^3-3x^2-18x$$

$$\underline{-\quad +\quad +\quad}$$

$$-10x^2+5x+30$$

$$-10x^2+5x+30$$

$$\underline{+\quad -\quad -\quad}$$

$$\times$$

∴ Answer = $3x - 5$

(xiv)

$$2a+3b+5c \overline{\smash{)}\ 4a^2+12ab+9b^2-25c^2}\ (2a+3b-5c$$

$$\underline{4a^2+6ab \hphantom{+12ab+9b^2-25c^2}+10ca}$$
$$-\ \ -\ \hphantom{+12ab+9b^2-25c^2}\ -$$

$$6ab+9b^2-25c^2\ -10ca$$
$$\underline{6ab+9b^2\hphantom{-25c^2}+15bc}$$
$$-\ \ -\ \hphantom{+9b^2-25c^2}\ -$$

$$-10ca-25c^2-15bc$$
$$\underline{-10ca-25c^2-15bc}$$
$$+\ \ \ +\ \ \ +$$
$$\overline{\hphantom{-10ca-25c^2}\times}$$

$\therefore$ Answer $= 2a+3b-5c$

(xv)

$$-x^3+x+4 \overline{\smash{)}\ +x^6-2x^4-8x^3+x^2+8x+16}\ (-x^3+x+4$$

$$\underline{+x^6-\ x^4-\ 4x^3}$$
$$-\ \ +\ \ +$$

$$-x^4-4x^3+x^2+8x+16$$
$$\underline{-x^4 \hphantom{-4x^3}+x^2+4x}$$
$$+\ \hphantom{-4x^3}\ -\ \ -$$

$$-4x^3+4x+16$$
$$\underline{-4x^3+4x+16}$$
$$+\ \ \ -\ \ \ -$$
$$\overline{\hphantom{-4x^3+}\times}$$

$\therefore$ Answer $= -x^3 + x + 4$

Question 2.

Find the quotient and the remainder (if any) when :

(i) $a^3 - 5a^2 + 8a + 15$ is divided by $a + 1$.

(ii) $3x^4 + 6x^3 - 6x^2 + 2x - 7$ is divided by $x - 3$.

(iii) $6x^2 + x - 15$ is divided by $3x + 5$.   In each case, verify your answer.

(iv) $6y^5 + 30y^4 + 18y^3 + 6y^2 + 15y + 3$ is divided by $2y^3 + 1$.

Solution:

(i)

$$a + 1 \overline{)\, a^3 - 5a^2 + 8a + 15 \,} \,(a^2 - 6a + 14$$

$$\underline{a^3 + a^2}$$
$$\quad - \quad -$$

$$-6a^2 + 8a + 15$$
$$\underline{-6a^2 - 6a}$$
$$\quad + \qquad +$$

$$14a + 15$$
$$\underline{14a + 14}$$

$$1$$

∴ Quotient $= a^2 - 6a + 14$ and remainder $= 1$

(ii)

$$x - 3 \overline{)\, 3x^4 + 6x^3 - 6x^2 + 2x - 7 \,}(3x^3 + 15x^2 + 39x + 119$$

$$\underline{3x^4 - 9x^3}$$
$$\quad - \quad +$$

$$15x^3 - 6x^2 + 2x - 7$$
$$\underline{15x^3 - 45x^2}$$
$$\quad - \qquad +$$

$$39x^2 + 2x - 7$$
$$\underline{39x^2 - 117x}$$
$$\quad - \qquad +$$

$$119x - 7$$
$$\underline{119x - 357}$$
$$\quad - \qquad +$$

$$350$$

∴ Quotient $= 3x^3 + 15x^2 + 39x + 119$ and remainder $= 350$

(iii) $3x + 5 \overline{)6x^2 + x - 15} (2x - 3$

$\quad\quad\quad 6x^2 + 10x$

$\quad\quad\quad \underline{\;-\quad--\;}$

$\quad\quad\quad\quad -9x - 15$

$\quad\quad\quad\quad -9x - 15$

$\quad\quad\quad\quad \underline{\;+\quad+\;}$

$\quad\quad\quad\quad\quad\quad\times$

$\therefore$ Quotient = $2x - 3$ and remainder = 0

(iv) $\quad 2y^3 + 1 \overline{)6y^5 + 30y^4 + 18y^3 + 6y^2 + 15y + 3} (3y^2 + 15y + 9$

$\quad\quad\quad 6y^5 \quad\quad\quad\quad\quad\quad + 3y^2$

$\quad\quad\quad \underline{\;-\quad\quad\quad\quad\quad---\;}$

$\quad\quad\quad\quad 30y^4 + 18y^3 + 3y^2 + 15y + 3$

$\quad\quad\quad\quad 30y^4 \quad\quad\quad\quad\quad\quad + 15y$

$\quad\quad\quad\quad \underline{\;-\quad\quad--\quad\quad\quad\;}$

$\quad\quad\quad\quad\quad\quad 18y^3 + 3y^2 + 3$

$\quad\quad\quad\quad\quad\quad 18y^3 \quad\quad\quad + 9$

$\quad\quad\quad\quad\quad\quad \underline{\;--\quad\quad\quad-\;}$

$\quad\quad\quad\quad\quad\quad\quad\quad 3y^2 - 6$

$\therefore$ Quotient = $3y^2 + 15y + 9$ and remainder = $3y^2 - 6$

(i) Verification.

Dividend = Quotient × Divisor + Remainder

$= (a^2 - 6a + 14) \times (a + 1) + 1$

$= a^3 - 6a^2 + 14a + a^2 - 6a + 14 + 1$

$= a^3 - 5a^2 + 8a + 15$ which is given

(ii) Verification:

Dividend = Quotient × Divisor + Remainder

$= (3x^3 + 15x^2 + 39x + 119)(x - 3) + 350$

$= 3x^4 + 15x^3 + 39x^2 + 119x - 9x^3 - 45x^2 -$

$117x - 357 + 350$

$= 3x^4 + 6x^3 - 6x^2 + 2x - 7$ which is given

(iii) Verification:

Dividend = Quotient × Divisor + Remainder

$= (2x - 3)(3x + 5) + 0$

$= 6x^2 + 10x - 9x - 15 + 0$

$= 6x^2 - x - 15$ which is given

(iv) Verification:

Dividend = Quotient × Divisor + Remainder

$= (3y^2 + 15y + 9)(2y^3 + 1) + 3y^2 - 6$

$= 6y^5 + 30y^4 + 18y^3 + 3y^2 + 15y + 9 + 3y^2 - 6$

$= 6y^5 + 30y^4 + 18y^3 + 6y^2 + 15y + 3$ which is given

Question 3.
The area of a rectangle is $x^3 - 8x^2 + 7$ and one of its sides is $x - 1$. Find the length of the adjacent side.

Solution:

$$\text{Area} = x^3 - 8x^2 + 7$$

$$\text{One side} = x - 1$$

$$\therefore \text{Adjacent side} = (x^3 - 8x^2 + 7) \div (x - 1)$$

$$
\begin{array}{r}
x^2 - 7x - 7 \\
x - 1\,\overline{)\,x^3 - 8x^2 + 7} \\
\underline{x^3 - x^2} \\
\;\;-\;\;\;+ \\
\hline
-7x^2 + 7 \\
-7x^2 + 7x \\
\;\;+\;\;\;- \\
\hline
-7x + 7 \\
-7x + 7 \\
\;\;+\;\;\;- \\
\hline
\times
\end{array}
$$

$$\therefore \text{Other side} = x^2 - 7x - 7$$

Question 4.
The product of two numbers-is $16x^4 - 1$. If one number is $2x - 1$, find the other.

Solution:

$$\text{Product of two numbers} = 16x^4 - 1$$

$$\text{One number} = 2x - 1$$

$$\text{Then second number} = \frac{16x^4 - 1}{2x - 1}$$

$$= 8x^3 + 4x^2 + 2x + 1$$

$$
\begin{array}{r}
8x^3 + 4x^2 + 2x + 1 \\
2x - 1\,\overline{)\,16x^4 \qquad\qquad\quad - 1} \\
\underline{16x^4 - 8x^3} \\
\;\;-\;\;\;+ \\
\hline
8x^3 \\
8x^3 - 4x^2 \\
\;\;-\;\;\;+ \\
\hline
4x^2 \\
4x^2 - 2x \\
\;\;-\;\;\;+ \\
\hline
2x - 1 \\
2x - 1 \\
\;\;-\;\;\;+ \\
\hline
\times
\end{array}
$$

Question 5.

Divide $x^6 - y^6$ by the product of $x^2 + xy + y^2$ and $x - y$.

Solution:

Product of $(x^2 + xy + y^2)$ and $(x - y)$

$= (x - y)(x^2 + xy + y^2)$

$= x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$

$= x^3 - y^3$

Now, $(x^6 - y^6) \div (x^3 - y^3)$

$= x^3 + y^3$

$$
\begin{array}{r}
x^3 + y^3 \\
x^3 - y^3 \overline{\smash{)}\, x^6 \qquad\quad - y^6} \\
x^6 - x^3y^3 \\
\underline{-\quad +} \\
x^3y^3 - y^6 \\
x^3y^3 - y^6 \\
\underline{-\quad +} \\
\times
\end{array}
$$

Simplification
(Using removal of brackets)
The signs for different types of brackets are :

1. ____ ; Vinculum or bar brackets,
2. ( ); Parenthesis or small brackets,
3. { }; Curly brackets or middle brackets,
4. [ ]; Square brackets or big brackets.
   In a combined operation, the brackets must be removed in the same order as written above:

Simplify :
Question 1.

$a^2 - 2a + \{5a^2 - (3a - 4a^2)\}$

Solution:

$= a^2 - 2a + \{5a^2 - 3a + 4a^2\}$
$= a^2 - 2a + \{9a - 3a\}$
$= a^2 - 2a + 9a^2 - 3a = 10a^2 - 5a$

Question 2.

$x - y - \{x - y - (x + y) - \overline{x - y}\}$

Solution:

$x - y - \{x - y - (x + y) - \overline{x - y}\}$
$= x - y - \{x - y - (x + y) - x + y\}$
$= x - y - \{x - y - x - y - x + y\}$
$= x - y - x + y + x + y + x - y = 2x$

Question 3.

$-3(1 - x^2) - 2\{x^2 - (3 - 2x^2)\}$

Solution:

$-3(1 - x^2) - 2\{x^2 - (3 - 2x^2)\}$
$= -3 + 3x^2 - 2\{x^2 - 3 + 2x^2\}$
$= -3 + 3x^2 - 2\{3x^2 - 3\}$
$= -3 + 3x^2 - 6x^2 + 6 = 3 - 3x^2$

Question 4.

$2\{m - 3(n + \overline{m - 2n})\}$

Solution:

$2\{m - 3(n + \overline{m - 2n})\}$
$= 2\{m - 3(n + m - 2n)\} = 2\{m - 3(m - n)\}$
$= 2\{m - 3m + 3n\} = 2\{3n - 2m\} = 6n - 4m$

Question 5.

$3x - [3x - \{3x - (3x - \overline{3x - y})\}]$

Solution:

$3x - [3x - \{3x - (3x - \overline{3x - y})\}]$
$= 3x - [3x - \{3x - (3x - 3x + y)\}]$
$= 3x - [3x - \{3x - y\}] = 3x - [3x - 3x + y]$
$= 3x - y$

Question 6.

$p^2x - 2\{px - 3x (x^2 - \overline{3a - x^2})\}$

Solution:

$p^2x - 2\{px - 3x (x^2 - \overline{3a - x^2})\}$

$= p^2x - 2\{px - 3x (x^2 - 3a + x^2)\}$

$= p^2x - 2\{px - 3x (2x^2 - 3a)\}$

$= p^2x - 2\{px - 6x^3 + 9ax\}$

$= p^2x - 2px + 12x^3 - 18ax$

Question 7.

$2[6 + 4\{m - 6(7 - \overline{n+p}) + q\}]$

Solution:

$2[6 + 4\{m - 6(7 - \overline{n+p}) + q\}]$

$= 2[6 + 4\{m - 6 (7 - n - p) + q\}]$

$= 2[6 + 4\{m - 42 + 6n + 6p + q\}]$

$= 2[6 + 4m - 168 + 24n + 24p + 4q]$

$= 2[4m + 24n + 24p + 4q - 162]$

$= 8m + 48n + 48p + 8q - 324$

Question 8.

$. a - [a - \overline{b+a} - \{a - (a - \overline{b-a})\}]$

Solution:

$a - [a - \overline{b+a} - \{a - (a - \overline{b-a})\}]$

$= a - [a - b - a - \{a - (a - b + a)\}]$

$= a - [- b - \{a - a + b - a\}]$

$= a - [- b - b + a]$

$= a + b + b - a = 2b$

Question 9.

$$3x - [4x - \overline{3x-5y} - 3\{2x - (3x - \overline{2x-3y})\}]$$

Solution:

$$3x - [4x - \overline{3x-5y} - 3\{2x - (3x -$$
$$\overline{2x-3y})\}]$$
$$= 3x - [4x - 3x + 5y - 3\{2x - (3x - 2x + 3y)\}]$$
$$= 3x - [4x - 3x + 5y - 3\{2x - (x + 3y)\}]$$
$$= 3x - [4x - 3x + 5y - 3\{2x - x - 3y\}]$$
$$= 3x - [x + 5y - 6x + 3x + 9y]$$
$$= 3x - [-2x + 14y]$$
$$= 3x + 2x - 14y$$
$$= 5x - 14y$$

Question 10.

$a^5 \div a^3 + 3a \times 2a$

Solution:

$a^5 \div a^3 + 3a \times 2a = a^{5-3} + 3a \times 2a = a^2 + 6a^2 = 7a^2$

Question 11.

$x^5 \div (x^2 \times y^2) \times y^3$

Solution:

$x^5 \div (x^2 \times y^2) \times y^3 = \dfrac{x^5}{x^2 y^2} \times y^3 = x^{5-2} - y^{3-2} = x^3 y$

Question 12.

$(x^5 \div x^2) \times y^2 \times y^3$

Solution:

$(x^5 \div x^2) \times y^2 \times y^3 = x^{5-2} \times y^{2+3} = x^3 y^5$

Question 13.

$(y^3 - 5y^2) \div y \times (y - 1)$

Solution:

$(y^3 - 5y^2) \div y \times (y - 1) = \dfrac{y^3 - 5y^2}{y} \times y - 1$

$= (y^2 - 5y) \times (y - 1) = y^2(y - 1) - 5y(y - 1) = y^3 - y^2 - 5y^2 + 5y = y^3 - 6y^2 + 5y$

Question 14.

$3a \times [8b \div 4 - 6 \{a - (5a - \overline{3b - 2a})\}]$

Solution:

$3a \times [8b \div 4 - 6 \{a - (5a - \overline{3b - 2a})\}]$

$= 3a \times \left[\dfrac{8b}{4} - 6\{a - (5a - 3b + 2a)\}\right] = 3a \times [2b - 6\{a - 7a + 3b\}] = 3a \times [2b - 6\{-6a + 3b\}]$

$= 3a \times [2b + 36a - 18b] = 3a \times [36a - 16b] = 108a^2 - 48ab$

Question 15.

$7x + 4\{x^2 \div (5x \div 10)\} - 3\{2 - x^3 \div (3x^2 \div x)\}$

Solution:

$7x + 4\{x^2 \div (5x \div 10)\} - 3\{2 - x^3 \div (3x^2 \div x)\}$

$= 7x + 4\left\{x^2 \div \left(\dfrac{5x}{10}\right)\right\} - 3\left\{2 - x^3 \div \left(\dfrac{3x^2}{x}\right)\right\}$

$= 7x + 4\left\{x^2 \div \dfrac{x}{2}\right\} - 3\left\{2 - x^3 \div 3x\right\} = 7x + 4\left\{x^2 \times \dfrac{2}{x}\right\} - 3\left\{2 - \dfrac{x^3}{3x}\right\}$

$= 7x + 8x - 6 + x^2 = x^2 + 15x - 6$